Central limit theorem for dependent multidimensionally indexed random variables

Tasos C. Christofides\textsuperscript{a,}\textsuperscript{*}, Petroula M. Mavrikiou\textsuperscript{b}

\textsuperscript{a}Dept. of Mathematics and Statistics, University of Cyprus, P.O. Box 20537, CY 1678 Nicosia, Cyprus
\textsuperscript{b}Dept. of Business Administration, Frederick Institute of Technology, P.O. Box 24729, CY 1303 Nicosia, Cyprus

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Abstract

We consider dependent multidimensionally indexed random variables whose dependence is determined by the distance of their indices. This provides a generalization of the well-known notion of \(m\)-dependence. For the partial sum of a collection of such variables we prove a central limit theorem.

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1. Introduction

One of the most widely studied subjects in probability theory is the concept of dependence. The nature of dependence varies and unless specific assumptions are made about the dependence between random variables, no meaningful statistical model can be assumed. A measure of dependence indicates how closely two random variables \(X\) and \(Y\) are, with extremes at mutual independence and complete mutual dependence. Measures of dependence could be conditions based on order or time between random variables, or could be conditions expressed in terms of a covariance or a correlation coefficient. Distance is also considered as a measure of dependence. For example, Fréchet (1946) proposed the use of an average of the distances of the distribution of \(Y\) conditional on \(X\) from some typical value such as the conditional mean or median. Fréchet (1948) also proposed the use of the Lévy metric (distance). A very important kind of dependence considering distance as a...